

Instability of brane cosmological solutions with flux compactifications

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We discuss the stability of the higher-dimensional de Sitter (dS) brane solutions with two-dimensional internal space in the Einstein-Maxwell theory. We show that an instability appears in the scalar-type perturbations with respect to the dS spacetime. We derive a differential relation which has the very similar structure to the ordinary laws of thermodynamics as an extension of the work for the six-dimensional model [21]. In this relation, the area of dS horizon (integrated over the two internal dimensions) exactly behaves as the thermodynamical entropy. The dynamically unstable solutions are in the thermodynamically unstable branch. An unstable dS compactification either evolves toward a stable configuration or two-dimensional internal space is decompactified. These dS brane solutions are equivalent to the accelerating cosmological solutions in the six-dimensional Einstein-Maxwell-dilaton theory via dimensional reduction. Thus, if the seed higher-dimensional solution is unstable, the corresponding six-dimensional solution is also unstable. From the effective four-dimensional point of view, a cosmological evolution from an unstable cosmological solution in higher dimensions may be seen as a process of the transition from the initial cosmological inflation to the current dark energy dominated Universe.

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I. INTRODUCTION

Six-dimensional braneworld models have attracted particular interests in recent years[1]. From the cosmological aspects, these models may be useful as a way of resolution of the cosmological constant problem, since a codimension two brane helps the sideslip of the brane vacuum energy into the bulk [2]. It has been pointed out that the original proposal of the resolution of this problem actually does not work well [3]. Nevertheless, six dimensional models has been recognized as an important playground to understand cosmology and gravity in higher-dimensional theory with non-trivial fluxes. The flux stabilization of extra dimensions would be a powerful tool to obtain realistic phenomenology and cosmology from string theory. In the simplest realization of the flux compactifications in six dimensions, the internal space has the shape of a rugby ball [2, 3], where codimension two branes are located at the positions of the poles. The warped generalizations of the rugby ball solutions have also been reported in the context of the six-dimensional Nishino-Sezgin (Salam-Sezgin), gauged supergravity [4] (see [5] for the original supergravity) and pure Einstein-Maxwell theory [6].¹

It also has been recognized that a 3-brane in six or higher dimensions generically have the problems on localization of matter on the brane due to its stronger self-gravity. One well motivated way to circumvent this problem is to regularize the brane, by taking the microscopic structure of the brane into account. Several ways of regularization of codimension two branes have been proposed in [8, 9, 10]. Based on these regularizations, low energy cosmology [10, 11, 12] and effective gravity on the brane [13] have been studied. On the other hand, the exact solutions [15, 16, 17, 18] will help to obtain unique observational/experimental predictions from six dimensions.

Stability of six-dimensional flux compactifications is an important issue and several analyses of linear perturbations have been reported. It has been reported that the Minkowski brane solutions in the supergravity [5] are marginally stable [19] and those in the Einstein-Maxwell theory are stable [20]. On the other hand, in the de Sitter (dS) brane solutions in the Einstein-Maxwell theory an instability appears in the scalar sector of perturbations with respect to the symmetry of dS spacetime for a relatively higher brane expansion rate [21]. This type of instability is commonly known in the dS spacetime [22] with an internal space compactified by a flux [24].² We will see that such an instability also appears in the dS brane solutions in higher-dimensional Einstein-Maxwell theory. The important fact is that a class of

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¹ In Ref. [7], a study on warped codimension-two braneworld solutions was presented on the analogy of the classical mechanics.

² Other instabilities in the quadrupole or higher multipole modes were reported, which gives rise to deformation of the internal space geometry, see e.g., Ref. [23]. However, these instabilities appear in dS compactifications with more than four-dimensional internal space and are not relevant for the models discussed in this paper.

the six-dimensional Einstein-Maxwell-dilaton theories has the equivalent structure to that of the higher-dimensional Einstein-Maxwell theory via dimensional reduction [16]. Thus instability of a solution in the higher dimensional theory suggests that of the corresponding solution in six-dimensional theory.

Also in Ref. [21], an important relation which has very similar structure to the ordinary laws of thermodynamics was found in the dS brane solutions in the six-dimensional Einstein-Maxwell theory. In this relation, the area of dS horizon (integrated over the internal space) exactly behaves as the usual thermodynamical entropy. It was shown that dynamically unstable solutions are also *thermodynamically* unstable. This may be seen as an example to support the conjecture that claims the equivalence of these two instabilities [26], which originally has been discussed for black brane solutions. We will see that such a thermodynamical relation can be easily extended to the case the higher dimensional dS brane solutions.

The dynamical evolution from unstable dS flux compactifications [24] have been investigated in e.g., Ref. [25]. Based on these arguments, we will discuss the fate of unstable dS brane solutions and corresponding cosmological solutions in six dimensions. We will see that an initially unstable dS brane solutions evolves to another stable dS or anti-de Sitter (AdS) brane solution unless the internal space is decompactified.

This article is organized as follows. In the section II, we review the higher-dimensional dS brane solutions and cosmological solutions in the general six-dimensional Einstein-Maxwell-dilaton theory via dimensional reduction from the dS brane solutions. In the section III, we analyze the stability and cosmological evolutions of the higher-dimensional dS compactifications. In the section IV, we discuss the possible fate of unstable solutions in the higher dimensional theory from the six-dimensional perspectives. In the section V, we shall close this article.

II. DE SITTER BRANE SOLUTIONS WITH FLUX COMPACTIFICATION

We start with the $(D + 2)$ -dimensional Einstein-Maxwell action

$$S^{(D+2)} = \int d^{D+2}X \sqrt{-G} \left[\frac{1}{2} \left({}^{(D+2)}R - 2\Lambda \right) - \frac{1}{4} {}^{(D+2)}F_{MN} {}^{(D+2)}F^{MN} \right], \quad (2.1)$$

and the D -dimensional brane actions contain the tension σ_i where $(i = 1, 2)$. We set the $(D + 2)$ -dimensional gravitational scale $M_{D+2} = 1$, unless it should be shown explicitly. There are the warped de Sitter (dS) brane solutions with two-dimensional internal space compactified by a magnetic flux. The bulk metric is given by

$$ds_{(D+2)}^2 = \xi^{(2-2\gamma^2)/(1+\gamma^2)} \gamma_{\alpha\beta} dz^\alpha dz^\beta + \frac{\xi^{-2\gamma^2/(1+\gamma^2)}}{2\Lambda} \left[\frac{d\xi^2}{h(\xi)} + \beta^2 h(\xi) d\theta^2 \right], \quad (2.2)$$

where

$$h(\xi) := \frac{(1+\gamma^2)^2}{4(5-\gamma^2)} \left[-\xi^{2/(1+\gamma^2)} + \frac{1-\alpha^{8/(1+\gamma^2)}}{1-\alpha^{(3+\gamma^2)/(1+\gamma^2)}} \frac{1}{\xi^{(3-\gamma^2)/(1+\gamma^2)}} - \frac{\alpha^{(3+\gamma^2)/(1+\gamma^2)}(1-\alpha^{(5-\gamma^2)/(1+\gamma^2)})}{1-\alpha^{(3+\gamma^2)/(1+\gamma^2)}} \frac{1}{\xi^{6/(1+\gamma^2)}} \right] \\ + \frac{\lambda}{\Lambda} \frac{1+\gamma^2}{6+2\gamma^2} \xi^{2\gamma^2/(1+\gamma^2)} \left[1 - \frac{1}{\xi^{(3+\gamma^2)/(1+\gamma^2)}} \right] \left[1 - \frac{\alpha^{(3+\gamma^2)/(1+\gamma^2)}}{\xi^{(3+\gamma^2)/(1+\gamma^2)}} \right], \quad (2.3)$$

has two positive roots at $\xi = 1, \alpha$. The D -dimensional metric $\gamma_{\alpha\beta}$ is that of dS spacetime, which satisfies $R_{\alpha\beta}[\gamma] = (2/(D-2))\lambda\gamma_{\alpha\beta}$ (The expansion rate with respect to the dS proper time is given by $H^2 = 2\lambda/(D-1)(D-2)$). In the above expression, we have also introduced

$$\gamma := \sqrt{\frac{D-4}{D}}. \quad (2.4)$$

As we will see later, γ is being the dilatonic coupling in a class of the Einstein-Maxwell-dilaton theories which are equivalent to the higher-dimensional Einstein-Maxwell theory via dimensional reduction (see Eq. (4.2)). After a dimensional reduction, D needs not to be restricted to be an integer. We restrict $\alpha \leq \xi \leq 1$ and assume that θ has the period 2π . The field strength is now given by

$${}^{(D+2)}F_{\xi\theta} = \frac{\beta}{\sqrt{2\Lambda}} \frac{Q}{\xi^{2(2+\gamma^2)/(1+\gamma^2)}} \quad (2.5)$$

where the magnetic charge Q is

$$Q := \alpha^{\frac{3+\gamma^2}{2(1+\gamma^2)}} \left(\frac{3+\gamma^2}{5-\gamma^2} \frac{1-\alpha^{(5-\gamma^2)/(1+\gamma^2)}}{1-\alpha^{(3+\gamma^2)/(1+\gamma^2)}} - \frac{2}{1+\gamma^2} \frac{\lambda}{\Lambda} \right)^{1/2}. \quad (2.6)$$

λ is bounded from above:

$$\lambda \leq \lambda_{\max}(\alpha) := \frac{(1+\gamma^2)(3+\gamma^2)}{2(5-\gamma^2)} \frac{1-\alpha^{(5-\gamma^2)/(1+\gamma^2)}}{1-\alpha^{(3+\gamma^2)/(1+\gamma^2)}} \Lambda. \quad (2.7)$$

The constant β controls deficit angles at $\xi = 1$ and $\xi = \alpha$, which are given, respectively, by $\delta_i = 2\pi[1-\beta|h'(\xi)|_{\xi=\xi_i}/2]$, where $i = +, -$ represents the branes $\xi_+ = 1$ or $\xi_- = \alpha$, respectively. The conical deficit corresponds to a codimension two brane and the tension is given by $\sigma_i/M_{D+2}^D = \delta_i$.

III. INSTABILITY OF DS BRANE SOLUTIONS

A. Dynamical instability

We briefly discuss the stability of the higher-dimensional dS brane solutions against the scalar-type perturbations with respect to the symmetry of D -dimensional dS spacetime. It is instructive to focus on the case $\alpha = 1$, where analytic solutions for perturbations are available. Introducing a new bulk coordinate $\xi = ((1-\alpha)\sin w + (1+\alpha))/2$, in order to resolve the spacetime structure in the limit $\alpha = 1$, the $(D+2)$ -dimensional metric Eq. (2.2) in the case $\alpha = 1$ can be expressed as

$$ds_{(D+2)}^2 = \gamma_{\mu\nu} dx^\mu dx^\nu + \frac{1+\gamma^2}{2\Lambda(1+\gamma^2) - (3+\gamma^2)\lambda} (dw^2 + \tilde{\beta}^2 \cos^2 w d\theta^2), \quad (3.1)$$

where $\tilde{\beta}$ is a constant related to β . The vector field is also rewritten into the form of $A_\theta \propto \sin w$.

We discuss scalar perturbations and work in the longitudinal gauge.

$$\begin{aligned} ds_{(D+2)}^2 &= (1 + 2\Omega_2(w, x^\mu)) \gamma_{\mu\nu} dx^\mu dx^\nu + \frac{1+\gamma^2}{2\Lambda(1+\gamma^2) - (3+\gamma^2)\lambda} \\ &\times \left[(1 + 2(\Omega_1(w, x^\mu) + \Omega_2(w, x^\mu))) dw^2 + (1 + 2(-\Omega_1(w, x^\mu) - \frac{3+\gamma^2}{1-\gamma^2}\Omega_2(w, x^\mu))) \tilde{\beta}^2 \cos^2 w d\theta^2 \right]. \end{aligned} \quad (3.2)$$

Note that we assume that the all the perturbation mode are axial symmetric and drop the dependence on the angular coordinate θ . Then, the magnetic field perturbation is written as

$$a_\theta^{(1)} = \frac{2((1+\gamma^2)\Lambda - 2\lambda)}{2(1+\gamma^2)\Lambda - (3+\gamma^2)\lambda} \left(- (2\Omega_1 + \frac{4}{1-\gamma^2}\Omega_2) + \frac{\Omega_{1,w}}{\tan w} \right). \quad (3.3)$$

We expand in terms of eigenmodes $\Omega_i = \sum_n \chi_n(x^\mu) \omega_{i,n}(w)$ ($i = 1, 2$), where $\square_D \chi_n(x^\mu) = \mu_n^2 \chi_n^\mu$, where μ_n represents the effective D -dimensional mass parameter of the n -th Kaluza-Klein (KK) mode and \square_D is d'Alembertian with respect to D -dimensional dS metric $\gamma_{\mu\nu}$. These two variables obey a couple of equations of motion. The solutions for the bulk mode ω_i are given in terms of the Legendre functions of order ν_\pm : $\nu_\pm(\nu_\pm + 1) = \lambda_\pm$, where

$$\lambda_\pm = 1 + \frac{(1+\gamma^2)\mu^2}{2(1+\gamma^2)\Lambda - (3+\gamma^2)\lambda} \pm \sqrt{1 + \frac{2(1+\gamma^2)(3+\gamma^2)((1+\gamma^2)\Lambda - 2\lambda)\mu^2}{(2(1+\gamma^2)\Lambda - (3+\gamma^2)\lambda)^2}}. \quad (3.4)$$

We impose the regularity at both the boundary branes and then we obtain the eigenmodes from the conditions $\nu_\pm = n(= 0, 1, 2)$. For the (+)-branch, the mass of the lowest mode is given by

$$\mu_0^2 = (1+\gamma^2) \left(1 - \frac{(3+\gamma^2)\lambda}{(1+\gamma^2)^2\Lambda} \right) \Lambda. \quad (3.5)$$

Clearly, for

$$\frac{\lambda}{\Lambda} > \frac{\lambda_{\text{ins}}}{\Lambda} := \frac{(1+\gamma^2)^2}{3+\gamma^2}, \quad (3.6)$$

the lowest mode becomes tachyonic. Note that the upper bound on the brane expansion rate is now given from Eq. (2.7)

$$\frac{\lambda_{\max}}{\Lambda} = \frac{1 + \gamma^2}{2}. \quad (3.7)$$

The lower tachyonic mass is bounded from the below

$$\mu_0^2 \geq -\frac{(1 - \gamma^2)\Lambda}{2}, \quad (3.8)$$

and hence this mode disappears in the limit $\gamma \rightarrow 1$ ($D \rightarrow \infty$), which corresponds to the Nishino-Sezgin (Salam-Sezgin) gauged supergravity theory in the equivalent six-dimensional picture (See Sec. IV).

Before closing this subsection, we shall comment on the stability against tensor- and vector-type perturbations with respect to the symmetry of the dS spacetime. Stability of the higher-dimensional dS brane solutions against the tensor perturbations has been shown in Ref. [16], irrespectively of α and λ . Note that stability of the six-dimensional solutions against the tensor perturbations was originally confirmed in Ref. [21]. In Ref. [21], stability against the vector perturbations was also shown. Both the stability against the tensor perturbations and the appearance of an instability against the scalar perturbations are the features that are not relevant for the number of dimensions of dS spacetime. Thus, it is quite natural to expect that the dS brane solutions are stable against the vector perturbations, irrespectively of the number of dimensions.

B. dS thermodynamics

In Ref. [21], for the dS brane solutions in the six-dimensional Einstein-Maxwell theory, a differential relation which has the very similar structure to the ordinary laws of thermodynamics has been derived. In this relation, the area of the dS horizon (cosmological horizon) integrated over the internal space behaves like the thermodynamical entropy. Then, it was shown that the dynamically unstable solutions were also *thermodynamically unstable*, namely these two instabilities were equivalent in such a system. As we will see in this subsection, the *dS thermodynamics* can be extended to the cases of higher dimensional dS brane solutions. The area of dS horizons (divided by the area of $(D - 2)$ -sphere Ω_{D-2}) is given by

$$\mathcal{A} = \frac{\beta H^{-(D-2)}}{(2\Lambda)} \frac{\pi(D-2)}{D-1} \left(1 - \alpha^{2(D-1)/(D-2)}\right). \quad (3.9)$$

We also find conserved quantities, the magnetic flux

$$\phi := \sqrt{2\Lambda} \int_{\alpha}^1 d\xi \int_0^{2\pi} d\theta^{(D+2)} F_{\xi\theta} = \frac{\pi\beta(D-2)}{(D-1)} \frac{Q}{\alpha^{2(D-1)/(D-2)}} \left(1 - \alpha^{2(D-1)/(D-2)}\right). \quad (3.10)$$

and

$$\beta_+ = \frac{-h_{,\xi}(\xi=1)}{2}\beta, \quad \beta_- = \frac{h_{,\xi}(\xi=\alpha)}{2}\beta, \quad (3.11)$$

which are directly related to the brane tensions, located at $\xi = 1$ and $\xi = \alpha$, as $\sigma_{\pm} = 2\pi(1 - \beta_{\pm})$. It is straightforward to find the relation

$$\beta_+ + \alpha^{2D/(D-2)}\beta_- = \frac{1}{2\pi} \left(Q\phi + (D-1)H^D \mathcal{A} \right). \quad (3.12)$$

Note that $H^2 = (2/(D-1)(D-2))\lambda$ is the expansion rate of the dS spacetime, with respect to the dS proper time.

The area of dS horizon is related to the area of dS horizon

$$S_E = - \int d^{D+2}X \sqrt{G} \left(\frac{1}{2}^{(D+2)} R - \Lambda - \frac{1}{4}^{(D+2)} F_{AB}^{(D+2)} F^{AB} \right) = -(D-1)\Omega_D \mathcal{A} \quad (3.13)$$

where Ω_D is the area of D -sphere. The stationary solutions correspond to the points where the Euclidean action has the maximum with respect to the variable (α, λ) . The observer at the $(+)$ -brane cannot adjust the tension of the $(-)$ -brane. It is useful to define the intensive quantities as $\tilde{\mathcal{A}} := \mathcal{A}/\beta_-$, $\tilde{\phi} := \phi/\beta_-$ and $\eta := \beta_+/\beta_-$. The similar quantities

divided by β_+ can be defined for the observer on $(-)$ -brane. Of course, these two points of views are equivalent. In the later discussion, we will take the point of view from the $(+)$ -brane. An extremal condition $(\partial S_E/\partial\alpha)_H = 0$ gives

$$\tilde{\phi} = 2\pi \left(\frac{2D}{D-1} \right) \frac{\alpha^{(D+2)/(D-2)}}{\left(\frac{\partial Q}{\partial \alpha} \right)_H}. \quad (3.14)$$

The other extremal condition $(\partial S_E/\partial H)_\alpha = 0$ gives

$$\left(H \left(\frac{\partial Q}{\partial H} \right)_\alpha - DQ \right) \phi = -2\pi D \left(\beta_+ + \beta_- \alpha^{2D/(D-2)} \right). \quad (3.15)$$

These two conditions determine the stationary points (α_e, H_e) , which are analytically continued to the stationary solutions in the original Lorentzian theory. They can be reduced to the following relations

$$\begin{aligned} (D-1)H_e^D \mathcal{A} &= 2\pi \left(\beta_+ + \beta_- \alpha_e^{2D/(D-2)} \right) - \phi Q_e \\ \phi dQ_e &= 2\pi \beta_- d\alpha_e^{2D/(D-2)} - (D-1)\mathcal{A} dH_e^D. \end{aligned} \quad (3.16)$$

The first one is the repetition of Eq. (3.12). From these relations, in terms of the *intensive* variables, the following differential relation is obtained

$$d(-\tilde{\mathcal{A}}) = \frac{1}{(D-1)H^D} \left(2\pi d(-\eta) + Q d\tilde{\phi} \right). \quad (3.17)$$

This relation has the same form as the first law of thermodynamics: For fixed β_- , we obtain

$$d(-\eta) = \frac{1}{2\pi\beta_-} d\sigma_+. \quad (3.18)$$

We can see that the change of $(-\eta)$ corresponds to that of the *internal energy* of the system. The magnetic flux $\tilde{\phi}$ and magnetic charge Q can be analogies of the *volume* and *pressure*, respectively. The dS expansion rate H^D can be seen as the thermodynamical *temperature*. So, $(-\tilde{\mathcal{A}})$ corresponds to the thermodynamical *entropy*.

For a fixed value of conserved quantity $(\tilde{\phi}, \eta)$, the area of a dS horizon is double-valued: there are two possible branches, thermodynamically unstable (low entropy) and stable (high entropy) branches. We can see that the dynamically unstable solutions are belonging to the low entropy branch. To show this, we consider the thermodynamical stability condition $\delta^2(-\mathcal{A}) < 0$: This requires the inequalities

$$\left(\frac{\partial \eta}{\partial H^D} \right)_Q < \left(\frac{\partial \eta}{\partial H^D} \right)_{\tilde{\phi}} < 0, \quad \left(\frac{\partial \tilde{\phi}}{\partial (QH^{-D})} \right)_H < \left(\frac{\partial \tilde{\phi}}{\partial (QH^{-D})} \right)_\eta < 0. \quad (3.19)$$

For instance, the first condition states that the *specific heats* are positive. We can explicitly obtain

$$\begin{aligned} \left(\frac{\partial \eta}{\partial H^D} \right)_{\tilde{\phi}} &= \frac{1}{\left(\frac{\partial \tilde{\phi}}{\partial \alpha} \right)_H} \frac{\partial(\eta, \tilde{\phi})}{\partial(H^D, \alpha)}, \\ \left(\frac{\partial(QH^{-D})}{\partial \tilde{\phi}} \right)_\eta &= -\frac{1}{H^{2D}} \left(\frac{\partial(\eta, \tilde{\phi})}{\partial(H^D, \alpha)} \right)^{-1} \left[\left(H^D \left(\frac{\partial Q}{\partial H^D} \right)_\alpha - Q \right) \left(\frac{\partial \eta}{\partial \alpha} \right)_H - H^D \left(\frac{\partial Q}{\partial \alpha} \right)_H \left(\frac{\partial \eta}{\partial H^D} \right)_\alpha \right]. \end{aligned} \quad (3.20)$$

It is straightforward to see that those quantities are negative for $\lambda < \lambda_{\text{crit}}(\alpha)$ for each α . Here $\lambda_{\text{crit}}(\alpha)$ represents the position of the critical curve, on which a map from (α, H) plane to $(\eta, \tilde{\phi})$ plane breaks down:

$$\frac{\partial(\eta, \tilde{\phi})}{\partial(\alpha, H)} = 0. \quad (3.21)$$

The critical curve can be analytically obtained (in terms of λ) as

$$\begin{aligned} \frac{\lambda_{\text{crit}}}{\lambda_{\text{max}}} &= \frac{1}{4(3+\gamma^2)(1-\alpha^{(5-\gamma^2)/(1+\gamma^2)})(1-\alpha^{(3+\gamma^2)/(1+\gamma^2)})^2} \\ &\times \left\{ 11 + 4\alpha^{-1+\frac{6}{1+\gamma^2}} - 25\alpha^{1+\frac{2}{1+\gamma^2}} - 4\alpha^{2+\frac{4}{1+\gamma^2}} + 25\alpha^{\frac{8}{1+\gamma^2}} - 11\alpha^{1+\frac{10}{1+\gamma^2}} + 6\gamma^2 - 4\alpha^{-1+\frac{6}{1+\gamma^2}}\gamma^2 \right. \\ &- 22\alpha^{1+\frac{2}{1+\gamma^2}}\gamma^2 + 4\alpha^{2+\frac{4}{1+\gamma^2}}\gamma^2 + 22\alpha^{\frac{8}{1+\gamma^2}}\gamma^2 - 6\alpha^{1+\frac{10}{1+\gamma^2}}\gamma^2 - \gamma^4 - \alpha^{1+\frac{2}{1+\gamma^2}}\gamma^4 + \alpha^{\frac{8}{1+\gamma^2}}\gamma^4 + \alpha^{1+\frac{10}{1+\gamma^2}}\gamma^4 \\ &\left. - \left(1 + 4\alpha^{-1+\frac{6}{1+\gamma^2}} - 4\alpha^{1+\frac{2}{1+\gamma^2}} - \gamma^2 - \alpha^{\frac{8}{1+\gamma^2}}(1-\gamma^2) \right) \sqrt{(1-\gamma^2)^2(1+\alpha^{2+\frac{4}{1+\gamma^2}}) + 2\alpha^{1+\frac{2}{1+\gamma^2}}(17+14\gamma^2+\gamma^4)} \right\}, \end{aligned} \quad (3.22)$$

where the maximum value of the flux λ_{\max} is given in Eq. (3.7). In the case that $\gamma \rightarrow 0$ (the six-dimensional limit), we recover the result in [21]

$$\frac{\lambda_{\text{crit}}}{\lambda_{\max}} = \frac{1}{12(1+\alpha+\alpha^2)(1+\alpha+\alpha^2+\alpha^3+\alpha^4+\alpha^5)} \left[11 + 33\alpha + 66\alpha^2 + 85\alpha^3 + 90\alpha^4 + 85\alpha^5 + 66\alpha^6 + 33\alpha^7 + 11\alpha^8 \right. \\ \left. - (1+\alpha)(1+2\alpha+4\alpha^2+2\alpha^3+\alpha^4)\sqrt{1+34\alpha^3+\alpha^6} \right]. \quad (3.23)$$

In Ref. [21], it has been seen that for the solutions belonging to the above critical curve the lowest mode of the scalar perturbations exactly becomes massless and thus this curve gives the border between the families of the stable and unstable solutions. This must be true for the higher dimensional dS solutions. For the general γ (hence D), the limit $\alpha \rightarrow 1$ gives

$$\frac{\lambda_{\text{crit}}}{\lambda_{\max}} = \frac{2(1+\gamma^2)}{3+\gamma^2}. \quad (3.24)$$

namely $\lambda_{\text{crit}} = \lambda_{\text{inst}}$, which is defined in Eq. (3.6).

C. Cosmological evolutions

In this subsection, we see the cosmological evolutions from an unstable dS solution. We assume that the D -dimensional geometry keeps homogeneity and isotropy after the deviation from the exact dS geometry. In our perturbation analysis in subsection III.A, we assume that Ω_1 and Ω_2 are functions of the space and time in D -dimensional dS space time (More precisely, we expanded them in terms of scalar harmonic functions defined on the D -dimensional dS spacetime). However, in order to discuss the cosmological evolutions, we assume the perturbations as the functions of only the time, in order to keep the homogeneity and isotropy of D -dimensional spacetime. This assumption will also be essential to discuss the cosmology in terms of the equivalent six-dimensional theory in Sec. IV.

We firstly focus on the case $\alpha = 1$. In this case, it is natural that the bulk shape is not deformed during cosmological evolution (and hence always $\alpha = 1$). We consider the homogeneous evolution of the external and internal space

$$ds^2 = -dt^2 + a(t)^2 d\mathbf{x}_{D-1}^2 + c(t)^2 \left[dw^2 + \tilde{\beta}^2 \cos^2 w d\theta^2 \right]. \quad (3.25)$$

For the conserved magnetic flux,

$$\phi = \sqrt{2\Lambda} \int dw d\theta^{(D+2)} F_{w\theta} = 4\pi\tilde{\beta} \frac{\sqrt{1 - \frac{\lambda}{\Lambda} \frac{D}{D-2}}}{1 - \frac{\lambda}{\Lambda} \frac{D-1}{D-2}} \quad (3.26)$$

the radion equation of motion is typically given by

$$\ddot{c} = - \left((D-1) \frac{\dot{a}}{a} + \frac{\dot{c}}{c} \right) \dot{c} - \frac{dV_{\text{eff}}(c)}{dc}, \quad (3.27)$$

where the effective potential is defined as

$$V_{\text{eff}}(c) = \frac{(3+\gamma^2)(\Lambda - \frac{2}{1+\gamma^2}\lambda_1)}{16(\Lambda - \frac{3+\gamma^2}{2(1+\gamma^2)}\lambda_1)^2 c^2} - \frac{(1-\gamma^2)\Lambda c^2}{4} + \ln(c) + \text{const}. \quad (3.28)$$

A typical example of the effective potential is shown in Fig. 1. There are local minimum at $c = c_2$ and local maximum at $c = c_1$, respectively. One possible evolution is that from an unstable configuration to a stable one, namely from the solution of the radius c_1 to that of c_2 . Assuming that correspondingly the brane curvature changes from λ_1 to λ_2 (We now allow for the possibility of $\lambda < 0$), the flux conservation gives the relation

$$\frac{\lambda_2}{\Lambda} = \frac{4(1+\gamma^2)^2}{(3+\gamma^2)^2} \frac{1 - \frac{(3+\gamma^2)^2}{4(1+\gamma^2)^2} \frac{\lambda_1}{\Lambda}}{1 - \frac{2}{1+\gamma^2} \frac{\lambda_1}{\Lambda}}. \quad (3.29)$$

Noting that $c_i = (2\Lambda - 2(D-1)\lambda_i/(D-2))^{-1/2}$, the initial and final radii are related as

$$\frac{c_2^2}{c_1^2} = \frac{3+\gamma^2}{1-\gamma^2} \left(1 - \frac{2}{1+\gamma^2} \frac{\lambda_1}{\Lambda} \right). \quad (3.30)$$

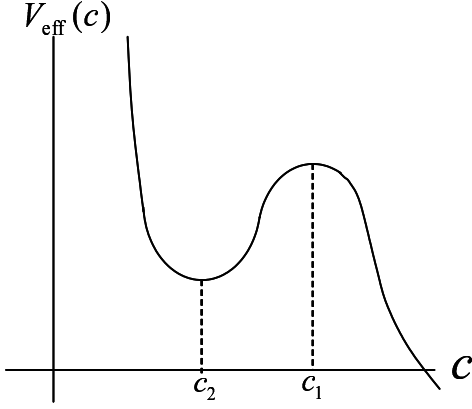


FIG. 1: The schematic view of the radion effective potential is shown. The local maximum and minimum are located at c_1 and c_2 , respectively. For $\lambda_1 > \lambda_{\text{inst}}$, $c_2 < c_1$ is always satisfied.

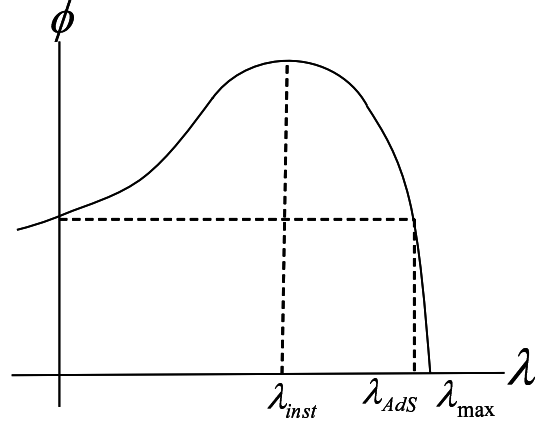


FIG. 2: A typical example of the behavior of the magnetic flux Eq. (3.26) is shown as a function λ/Λ . The sequence can be analytically continued to negative λ , i.e. AdS solutions. Around the maximum value, for the same value of the flux, there are two possible values of positive λ , namely unstable and stable solutions, implying the evolution between two solutions with conserved flux.

Clearly for $\lambda_1 > \lambda_{\text{inst}}$, $c_2 < c_1$. Thus, we see that the radius of two-dimensional internal space must become smaller during the evolution, if the unstable configuration evolves to the stable one. There is other possibility of cosmological evolution: the *decompactification* $c \rightarrow \infty$, namely to the $(D+2)$ -dimensional dS solution with expansion rate given by $(2\Lambda/D(D+1))^{1/2}$. These two possibilities may be distinguished in terms of the initial conditions. If initially $\dot{c} < 0$ the radion evolves to the true minimum at $c = c_2$, whereas it may evolve to $c \rightarrow \infty$, if initially $\dot{c} > 0$. We focus on more realistic the first possibility.

There is also another critical brane expansion rate

$$\frac{\lambda_{\text{AdS}}}{\Lambda} = \frac{4(1+\gamma)^2}{(3+\gamma)^2}. \quad (3.31)$$

Note that always $\lambda_{\text{inst}} < \lambda_{\text{AdS}} < \lambda_{\text{max}}$. For $\lambda_{\text{inst}} < \lambda_1 \leq \lambda_{\text{AdS}}$, $\lambda_2 \geq 0$ and thus the final D -dimensional geometry is dS, while for $\lambda_{\text{AdS}} < \lambda_1 < \lambda_{\text{max}}$, $\lambda_2 < 0$ and the final stable D -dimensional geometry is anti-de Sitter (AdS). The thermodynamical arguments cannot be applied to AdS brane configurations since we should require the reality of the *temperature* H^D (non-negativity of λ). In Fig. 2, we showed a typical example of the magnetic flux as a function of λ . As is seen, for increasing λ , the flux takes a maximum value at $\lambda = \lambda_{\text{inst}}$, then starts to decrease and vanishes at λ_{max} . For the values of $\lambda_{\text{inst}} < \lambda \leq \lambda_{\text{AdS}}$, an unstable solution evolve to another dS configuration. For the $\lambda_{\text{AdS}} < \lambda < \lambda_{\text{max}}$, the corresponding stable configuration is AdS.

The above discussion can be naturally applied to more general cases $\alpha < 1$. The observer on the (+)-brane cannot change the tension of (-)-brane and thus, ϕ/β_- , i.e. $\tilde{\phi}$ defined in the previous subsection is conserved during the cosmological evolution. Oppositely for the observer on the (-)-brane and ϕ/β_+ is conserved during the evolution. In the case $\alpha = 1$, the critical expansion rate where dynamical instability appears is the exactly the point given by $(\partial\tilde{\phi}/\partial H^D)_\alpha = 0$. In general, since

$$\left(\frac{\partial\tilde{\phi}}{\partial H^D}\right)_\eta = \left(\frac{\partial\tilde{\phi}}{\partial H^D}\right)_\alpha + \left(\frac{\partial\tilde{\phi}}{\partial\alpha}\right)_H \left(\frac{\partial\alpha}{\partial H^D}\right)_\eta = \left(\frac{\partial\tilde{\phi}}{\partial H^D}\right)_\alpha - \left(\frac{\partial\tilde{\phi}}{\partial\alpha}\right)_H \frac{\left(\frac{\partial\eta}{\partial H^D}\right)_\alpha}{\left(\frac{\partial\eta}{\partial\alpha}\right)_H}, \quad (3.32)$$

$(\partial\tilde{\phi}/\partial H^D)_\eta \neq (\partial\tilde{\phi}/\partial H^D)_\alpha$. The special case is that of $\alpha = 1$, where $(\partial\eta/\partial H^D)_\alpha \rightarrow 0$ and hence $(\partial\tilde{\phi}/\partial H^D)_\eta = (\partial\tilde{\phi}/\partial H^D)_\alpha$. The condition $(\partial\tilde{\phi}/\partial H^D)_\alpha = 0$ gives another curve on the (α, λ) plane as

$$\frac{\lambda_*}{\Lambda} = \frac{\alpha^{-2+2/(1+\gamma^2)}(1+\gamma^2)}{2(1-\alpha^{1+2/(1+\gamma^2)})(5-\gamma^2)} \times \left[2\alpha^{1+4/(1+\gamma^2)}(1-\gamma^2) + (-3\alpha^3 + \alpha^{2\gamma^2/(1+\gamma^2)} + 2\alpha^{4+2/(1+\gamma^2)})(3+\gamma^2) - \alpha^{2+6/(1+\gamma^2)}(5-\gamma^2) \right]. \quad (3.33)$$

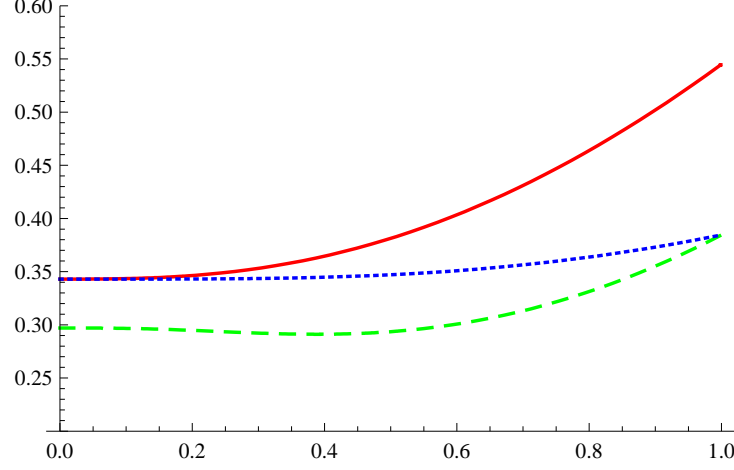


FIG. 3: λ_{\max} , λ_{crit} and λ_* (divided by Λ) are shown as a function of α for $\gamma = 0.3$, by solid (red), dashed (green) and dotted (blue) lines, respectively. The vertical and horizontal axes show λ and α , respectively. Note that the solutions in the region $\lambda_{\text{crit}} < \lambda < \lambda_{\max}$ are thermodynamically (also dynamically) unstable and λ_* is always located in this region.

Note that as special limits

$$\left. \frac{\lambda_*}{\Lambda} \right|_{\gamma \rightarrow 0} = \frac{3 + 6\alpha + 9\alpha^2 + 6\alpha^3 + 3\alpha^4 + 2\alpha^5 + \alpha^6}{10(1 + \alpha + \alpha^2)^2}, \quad \left. \frac{\lambda_*}{\Lambda} \right|_{\alpha \rightarrow 1} = \frac{(1 + \gamma^2)^2}{3 + \gamma^2}, \quad (3.34)$$

and hence $\alpha \rightarrow 1$ $\lambda_{\text{inst}} = \lambda_{\text{crit}} = \lambda_*$ as we have seen previously. It supports that in the case $\alpha = 1$, the cosmological evolution keeps the rugby ball shape during the cosmological evolution. However, in the cases $\alpha < 1$, $\lambda_{\text{crit}} \neq \lambda_*$ and hence it implies that the degree of warping α' of the final stable configuration is different from that of the initial one α in general.

Before closing this section, in Fig. 3, we showed an example of λ_{\max} , λ_{crit} and λ_* on the (α, λ) plane. As we see, in the limit $\alpha \rightarrow 1$, λ_{crit} and λ_* coincide while in the opposite limit $\alpha \rightarrow 0$, λ_* agrees with λ_{\max} . Thermodynamically unstable region corresponds to the region $\lambda_{\text{crit}} < \lambda < \lambda_{\max}$ for each α .

IV. COSMOLOGY IN SIX DIMENSIONS

A. Cosmological solutions in six dimensions

A class of six-dimensional Einstein-Maxwell-dilaton system has equivalent structure to the $(D + 2)$ -dimensional Einstein-Maxwell theory via dimensional reduction. We rewrite the $(D + 2)$ -dimensional metric as

$$ds_{(D+2)}^2 = \underbrace{e^{-(D-4)\phi(x)/2} g_{ab}(x) dx^a dx^b}_{6\text{D}} + \underbrace{e^{2\phi(x)} \delta_{mn} dy^m dy^n}_{(D-4)\text{D}}, \quad (4.1)$$

where $(D - 4)$ -dimensional part is compactified and the field strength with ${}^{(D+2)}F_{ab} = {}^{(D+2)}F_{ab}(x)$ and ${}^{(D+2)}F_{mM} = 0$. With the above ansatz, dimensional reduction on the $(D - 4)$ -dimensional manifold yields

$$S^{(6)} = \int d^6x \sqrt{-g} \left[\frac{1}{2} \left(R - \partial_a \varphi \partial^a \varphi - 2e^{-\gamma \varphi} \Lambda \right) - \frac{1}{4} e^{\gamma \varphi} F_{ab} F^{ab} \right], \quad (4.2)$$

with identifications: Eq. (2.4),

$$\varphi := \frac{\sqrt{(D-4)D}}{2} \phi, \quad (4.3)$$

and $F_{ab} := {}^{(D+2)}F_{ab} V_{D-4}^{1/2}$ (V_{D-4} is the volume of the $(D-4)$ -dimensional flat space). Here F_{ab} denotes the electromagnetic field strength. Again we set $M_6^4 := M_{D+2}^D V_{D-4} = 1$, unless it should be shown explicitly. For $\gamma = 1$ the action (4.2) coincides with the bosonic part of Nishino-Sezgin (Salam-Sezgin) supergravity (some fields are set to be zero consistently). From Eq. (2.4), $0 \leq \gamma \leq 1$. After dimensional reduction, D is a parameter related to the dilatonic coupling γ as Eq. (2.4), and hence can be arbitrary real, non-negative number. In particular, Nishino-Sezgin supergravity ($\gamma = 1$) [5] is reproduced by taking $D \rightarrow \infty$.

We also define the 3-brane tension as $\tau_i := \sigma_i V_{D-4}$. The tension of the 3-brane does not couple to the dilatonic scalar field after dimensional reduction. According to the above-mentioned way, we derive the cosmological solutions in the six-dimensional theory Eq. (4.2) from the higher-dimensional dS brane solutions Eq. (2.2). We identify $e^\phi = \xi^{(1-\gamma^2)/(1+\gamma^2)} e^{Ht}$. Then the six-dimensional metric is found to be

$$g_{ab} dx^a dx^b = \xi^{2/(1+\gamma^2)} [-d\tau^2 + a^2(\tau) \delta_{ij} dx^i dx^j] + \frac{b^2(\tau)}{2\Lambda} \left[\frac{d\xi^2}{h(\xi)} + \beta^2 h(\xi) d\theta^2 \right], \quad (4.4)$$

where the proper time is defined by $\tau = \int e^{(D-4)Ht/4} dt$, and the scale factors $a(\tau) := e^{DHt/4}$ and $b(\tau) := e^{(D-4)Ht/4}$, respectively. We easily find that $a(\tau) \propto \tau^{1/\gamma^2}$ and $b(\tau) \propto \tau$, leading to an accelerating cosmological solution for $\gamma < 1$. The case $\gamma = 1$ is exactly the (extensions) scaling solution in supergravity [15, 16]. We also find $\varphi(\tau, \xi) = (2/\gamma) \ln b(\tau) + (2\gamma)/(1+\gamma^2) \ln \xi$ and $F_{\xi\theta} = (\beta/\sqrt{2}) (Q/\xi^{2(2+\gamma^2)/(1+\gamma^2)})$, respectively. Other than this simplest solution there are two analytically known cosmological solutions derived from Kasner-type generalizations of $(D+2)$ dimensional dS brane solutions, but in the later times both these two solutions approach the above simplest solution [16]. Hence the above power-law solution is the late time attractor.

B. Cosmological implications

As we mentioned in the previous section, the class of the Einstein-Maxwell-dilaton theory Eq. (4.2) has the equivalent structure to the higher-dimensional Einstein-Maxwell theory via dimensional reduction. A $(D+2)$ -dimensional dS brane solution gives rise to the accelerating expanding cosmological solution. Thus, if a dS brane solution in $(D+2)$ dimensions is unstable, the corresponding cosmological solution given by Eq. (4.4) is also unstable. Since we assumed that the D -dimensional geometry keeps the Friedmann-Robertson-Walker form and hence there are no excitations of any perturbation mode which depends on the $(D-4)$ -dimensional spatial dimensions, the smooth one-to-one correspondence of the $(D+2)$ -dimensional Einstein-Maxwell theory with the six-dimensional Einstein-Maxwell-dilaton theory always exists at all the times of cosmological evolutions. If there is a trajectory in the solution space in the $(D+2)$ -dimensional theory, there also is a corresponding trajectory in the six-dimensional theory. It is also clear that the magnetic flux is conserved for the corresponding trajectory in six dimensions. The mass of an unstable mode scales as $M_0^2 \propto \mu_0^2/b(\tau)^2 \propto \mu_0^2/\tau^2$ from the six dimensional point of view. If the final stable solution in $(D+2)$ dimensional theory is dS brane solution, then the corresponding cosmological solutions in six-dimensions is also an accelerating cosmological solution with smaller λ . If the final configuration in $(D+2)$ -dimensions is AdS, then the corresponding six-dimensional solution would be a collapsing Universe, though there may be no analytic form since there is analytic description of AdS spacetime with the form of a flat Friedmann-Robertson-Walker metric.

The meaning of λ for a four-dimensional observer becomes clearer from the perspectives of the four-dimensional effective theory. From the original $(D+2)$ -dimensional metric

$$ds_{(D+2)}^2 = \xi^{2(1-\gamma^2)/(1+\gamma^2)} \left(\underbrace{e^{-(D-4)B(x^\mu)/2} q_{\mu\nu}(x) dx^\mu dx^\nu}_{4D} + \underbrace{e^{2B(x^\mu)} \delta_{mn} dy^m dy^n}_{(D-4)D} \right) + \frac{\xi^{-2\gamma^2/(1+\gamma^2)}}{2\Lambda} \left[\frac{d\xi^2}{h(\xi)} + \beta^2 h(\xi) d\theta^2 \right] \quad (4.5)$$

and magnetic field given in Eq. (2.5), the four-dimensional effective theory, composed of the four-dimensional metric $q_{\mu\nu}(x^\mu)$ and the modulus $B(x^\mu)$, for the observer on the (+)-brane is obtained. Then, after a conformal transformation $\hat{q}_{\mu\nu} = e^{2\gamma^2 B/(1-\gamma^2)} q_{\mu\nu}$, we can go to the Einstein frame. By defining the canonically normalized modulus $\chi := (2\gamma\sqrt{2(1+\gamma^2)/(1-\gamma^2)})B$, we obtain the four-dimensional effective Einstein-scalar theory with an exponential potential

$$S_{\text{eff}} = \int d^4x \sqrt{-\hat{q}} \left[\hat{R} - \frac{1}{2} \hat{q}^{\mu\nu} \partial_\mu \chi \partial_\nu \chi - 2\lambda e^{-\sqrt{2\gamma^2/(1+\gamma^2)}\chi} \right]. \quad (4.6)$$

Thus, λ characterizes the potential of an effective quintessential scalar field. The cosmic scale factor in the Einstein frame $\hat{a} \propto a^{1+\gamma^2}$ is proportional to $\hat{\tau}^{(1+\gamma^2)/(2\gamma^2)}$, where $\hat{\tau}$ is the cosmic proper time in the Einstein frame. This

scalar field may be the dark energy source, which accelerates the current Universe. A cosmological evolution in higher dimensions leads the geometry to stable solution with a smaller λ . The cosmological evolution in higher dimensions may be seen as a process of the transition from the initial cosmological inflation to the current dark energy dominated Universe from the four-dimensional perspectives.

V. SUMMARY

We discussed the stability of a de Sitter (dS) brane solutions in the higher-dimensional Einstein-Maxwell theory. We confirmed that the dS brane solutions with relatively high expansion rates are unstable against scalar-type linear perturbations with respect to the symmetry of dS spacetime. Such an instability commonly appears in the wide class of compactifications with an external dS spacetime and can be understood as a type of radionic instabilities.

Then, we generalized a relation found in the six-dimensional dS brane solutions Ref. [21], which has very similar structure to the ordinary laws of thermodynamics, to the case of the higher dimensions as Eq. (3.17). In this relation, the area of dS horizon (integrated over two internal dimensions) behaves the thermodynamical entropy. The area of dS horizon is essentially double-valued function of the flux and (one of) the brane tensions. A dynamically unstable solution is also in the thermodynamically unstable branch. The boundary between the thermodynamically stable and unstable branches is given by a curve, where one-to-one map from the plane of model parameters (α, λ) , where α controls the degree of warp and λ characterizes the curvature of dS spacetime, to the conserved quantities breaks down. These are the generalizations of the results of the case of six dimensions discussed in [21] to higher dimensions.

Then, we discussed the possible cosmological evolutions. We firstly focused on the case $\alpha = 1$, the case of an exactly rugby ball shaped bulk. In this case, during the cosmological evolution, the bulk keeps the rugby-ball type shape. There are two possibilities of the evolutions of unstable cosmological solutions: One possibility is to settle down a stable configuration. The other one is that the compactified two extra-dimensions are decompactified. This strongly depends on the initial condition and if the internal space is initially shrinking, the final configuration is another stable dS brane solution. Then, the size of the internal space in the final configuration usually is smaller than the initial unstable configuration. Furthermore, if initially $\lambda_{\text{inst}} < \lambda < \lambda_{\text{AdS}}$, where λ_{inst} and λ_{AdS} are given in Eq. (3.6) and Eq. (3.31), the final configuration is also a dS brane solution, while $\lambda_{\text{AdS}} < \lambda < \lambda_{\text{max}}$, the final configuration is an AdS brane solution. This picture can be easily generalized to the case of unstable dS configurations with non-trivial magnetic flux. In this case of the warped bulk $\alpha < 1$, the basic behavior remains the same as the rugby ball case: the initially unstable dS brane solution evolves toward the stable dS/AdS brane solutions (or decompactified). For the observer on the (+)-brane, during the cosmological evolution ϕ/β_- is conserved while for the observer on (-)-brane, ϕ/β_+ is conserved. In general, the shape of the bulk of the final stable configuration is different from that of the initial stable one.

Finally, we have discussed the stability of brane cosmological solutions in the six-dimensional Einstein-Maxwell-dilaton theory Eq. (4.2), including the Nishino-Sezgin (Salam-Sezgin), gauged supergravity as the special limit. This class of six-dimensional Einstein-Maxwell-dilaton theory has equivalent structure to the $(D+2)$ -dimensional Einstein-Maxwell theory and the number of dimensions D affects the dilatonic coupling in the reduced six-dimensional theory. Thus, if the seed dS brane solution is unstable in $(D+2)$ -dimensional spacetime, the corresponding cosmological solution in six dimensions is also unstable: if the final configuration is another dS brane solution in $(D+2)$ dimensions, this also may be true in six dimensions. The cosmological evolution in higher dimensions may be seen as a process of the transition from the initial cosmological inflation to the current dark energy dominated Universe from the four-dimensional perspectives.

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